

SOLVING TRIG EQUATIONS BY FACTORISING

A) Common Factors

Example: Solve the equation $2 \cos^2 x - \cos x = 0$ for $0^\circ \leq x \leq 360^\circ$.

$$\begin{aligned}2 \cos^2 x - \cos x &= 0 \\ \cos x (2 \cos x - 1) &= 0\end{aligned}$$

$$\text{So } \cos x = 0 \quad \text{or} \quad 2 \cos x - 1 = 0$$

Solving $\cos x = 0$ for $0^\circ \leq x \leq 360^\circ : x = 90^\circ, 270^\circ$.

Solving $2 \cos x - 1 = 0$ for $0^\circ \leq x \leq 360^\circ : x = 60^\circ, 300^\circ$.

So the solutions are $90^\circ, 270^\circ, 60^\circ, 300^\circ$.

B) Quadratic Factors

Examples:

1) Solve $2 \cos^2 x - 3 \cos x + 1 = 0$ for $0 \leq x \leq 2\pi$.

$$\begin{aligned}2 \cos^2 x - 3 \cos x + 1 &= 0 \\ (2 \cos x - 1)(\cos x - 1) &= 0\end{aligned}$$

$$\text{So } 2 \cos x - 1 = 0 \quad \text{or} \quad \cos x - 1 = 0$$

Solving $2 \cos x - 1 = 0$ for $0 \leq x \leq 2\pi : x = \frac{\pi}{3} \text{ or } 1.047, \frac{5\pi}{3} \text{ or } 5.236$

Solving $\cos x - 1 = 0$ for $0 \leq x \leq 2\pi : x = 0, 2\pi$

So the solutions are $\frac{\pi}{3}, \frac{5\pi}{3}, 0, 2\pi$.

2) Solve $2 \cos 2x + 7 \cos x = 0$ for $0^\circ \leq x \leq 180^\circ$.

$$\begin{aligned}2 \cos 2x + 7 \cos x &= 0 \\2(2 \cos^2 x - 1) + 7 \cos x &= 0 \\4 \cos^2 x - 2 + 7 \cos x &= 0 \\4 \cos^2 x + 7 \cos x - 2 &= 0 \\(4 \cos x - 1)(\cos x + 2) &= 0\end{aligned}$$



Using $\cos 2x = 2 \cos^2 x - 1$. This identity was chosen instead of the other two to keep all the terms in the equation cos.

$$\text{So } 4 \cos x - 1 = 0 \quad \text{or} \quad \cos x + 2 = 0$$

Solving $4 \cos x - 1 = 0$ for $0^\circ \leq x \leq 180^\circ : x = 75.52^\circ$.

Solving $\cos x + 2 = 0$ for $0^\circ \leq x \leq 180^\circ : \text{no solutions}$.

So the solution is 75.52° .

Delta Ex 35.6 pg 344 Q 1b, 3, 4a, 4c, 4e, 4f

Extension: Q5 – 9